Cartesian product of Sets.

Definition:—

Ordered pair:— An element of the form (x,y) is known as ordered pair where x,y belongs to i.e. any given set A.

The element x is called first element and y is called second element respectively.

i.e. $(x,y) \neq (y,x)$.

Cartesian product of two sets:— Let X and Y be any two sets. Let (x,y) be ordered pair, then cartesian product of X and Y denoted by $X \times Y$ (read as X cross Y and defined as

$X \times Y = \{(x,y) : x \in X, y \in Y)\}$ i.e. the first element of ordered pair belongs to the first set and second element of ordered pair belongs to the second set.

Cartesian product is general form:— Let $A_1, A_2, A_3, --- A_n$ be n given sets. The set of ordered n-triples $(a_1, a_2, a_3, --- a_n)$, $a_i \in A_i$ for $i = 1, 2, 3, ---, n$ is called the Cartesian product $A_1, A_2, A_3, --- , A_n$ and is denoted by $A_1 \times A_2 \times A_3 \times ---- \times A_n$.

~~Some Important definition~~

Cartesian product of an indexed family of sets:-

Let $(A_i : i \in I)$ be an indexed family of non-empty set A_i. Then the Cartesian product of this family denoted by $\underset{i\in I}{\cup} A_i$ is defined to the set of all functions $(x : \underset{i\in I}{\cup} A_i)$ defined over I and taking values is $\underset{i\in I}{\cup} A_i$ such that $x_i \in A_i$ for each $i \in I$.

Theorem 1:- Show that $A \subseteq B \Rightarrow A\times A = (A\times B)\cap(B\times A)$

Proof:- Given that $A \subseteq B \Rightarrow A\cap B = A$ —— (i)

We have $A\times A = (A\cap B)\times(A\cap B)$ from (1)

$= (A\cap B)\times(B\cap A)$ (By commutative law)

But $(A\times B)\cap(C\times D) = (A\cap C)\times(B\cap D)$

$\Rightarrow (A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)$

Replace C by B and D by A, we get

$(A\cap B)\times(B\cap A) = (A\times B)\cap(B\times A)$

$\Rightarrow A\times A = (A\times B)\cap(B\times A)$ (using (i) for $A\cap B = A$)

Proved.